$\therefore a^2+b^2+c^2+d^2+e^2+f^2=4[e^2+f^2+d^2]$;

$\therefore a^2+b^2+c^2=3[e^2+f^2+d^2]$.

Since $e^2+f^2+d^2$ (by construction) $=\dfrac{AC^2}{4}+\dfrac{CB^2}{4}+\dfrac{AB^2}{4}$

$\therefore a^2+b^2+c^2=\frac{3}{4}[AC^2+CB^2+AB^2]$.

Also solved by G. B. M. Zerr, J. Scheffer, and the Proposer.

CALCULUS.

255. Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Find the general values of u and v in terms of x, which satisfy the equations $u^2+l^2(du/dx)^2=v^2$, $u^2+m^2(du/dx)^2=v^2+n^2(dv/dx)^2$.

Solution by A. F. CARPENTER, Professor of Mathematics, Hastings, Neb.; GEORGE W. HARTWELL, Columbia University, and the PROPOSER.

Subtracting the first equation from the second, we get

$$[m^2-l^2]/n^2[du/dx]^2=[dv/dx]^2.$$

$$\therefore v=\left(\frac{m^2-l^2}{n^2}\right)^{\frac{1}{2}}u+C\ldots(1).$$

$$\therefore u^2+l^2[du/dx]^2=\frac{m^2-l^2}{n^2}u^2+2u\,C\left(\frac{m^2-l^2}{n^2}\right)^{\frac{1}{2}}+C^2.$$

Let $[m^2-l^2-n^2]/n^2=a$, and $C\left(\dfrac{m^2-l^2}{n^2}\right)^{\frac{1}{2}}=b$.

Then $l^2[du/dx]^2=au^2+2bu+C^2$.

$$\therefore x=l\int\frac{du}{\sqrt{[au^2+2bu+C^2]}}=\frac{l}{2C}\log\left(\frac{au+b-C}{au+b+C}\right)+\log C_1.$$

$$\therefore x=\frac{l}{2C}\log\left(\frac{C_1[au+b-C]}{au+b+C}\right).\quad \text{Let } e^{2Cx/l}=c.$$

Then $c/C_1=\dfrac{au+b-C}{au+b+C}$, or $u=\dfrac{[b+C]c-[b-C]C_1}{a[C_1-c]}$.

$$\therefore u=\frac{\{C[(m^2-l^2)^{\frac{1}{2}}+n]/n\}e^{2Cx/l}-C_1C[(m^2-l^2)^{\frac{1}{2}}-n]/n]}{[m^2-l^2-n^2][C_1-e^{2Cx/l}]/n^2}$$

$$=\frac{Cn\{[(m^2-l^2)^{\frac{1}{2}}+n]e^{2Cx/l}-C_1[(m^2-l^2)^{\frac{1}{2}}-n]\}}{[m^2-l^2-n^2][C_1-e^{2Cx/l}]}.$$

v is found at once from (1).

Also solved by V. M. Spunar, J. Scheffer, A. H. Holmes, and J. I. Wodo.